EXPERIMENTAL CHARACTERIZATION OF COMPOSITE MATERIALS

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Contents

Summary

Composite materials (Fiber Reinforced Polymers) offer light weight and high stiffness/strength. Due to these attributes their use in structural applications has increased dramatically practically at every length scale. A successful design and analysis of a composite structure requires knowledge of the material properties such as strength and stiffness. Material properties are determined in general by experimental methods. In the event the material properties of the composite are determined by

micromechanics models utilizing the properties of the constituents, then experimental determination of the constituent properties and experimental verification of the material property is required. As a background, a brief introduction to the mechanics of composite materials is presented which is then followed by the discussion of the experimental methods to determine the basic properties of a typical fiber reinforced polymer.

1. Introduction

Lightweight structures that meet even exceed the performance and safety requirements in the strictest sense are gaining more importance. Fiber reinforced polymer composites have been shown to perform successfully in many structural applications, and it is expected their use will increase further in the upcoming years.

A typical composite material consists of at least two distinct phases when combined offer advantages which cannot be offered by any one of its constituents alone. A composite material is sought after when lightweight structures are needed with sufficient strength and stiffness. Application of composite materials can be found from aerospace, marine, automotive areas to biomedical implants.

Composite materials fall into the general category of anisotropic materials, for which the material properties exhibit directional characteristics. On the other hand the most common engineering materials such as steel etc., are considered isotropic for which there is no dependence of material properties on direction. Isotropic materials can be characterized by two independent material constants only, but for anisotropic materials the number of constants can be as high as 21depending on the number of planes of material symmetry the material possesses.

2. Basics of Composite Materials

2.1. Constitutive Relationships

The constitutive relationship for the anisotropic materials is obtained through generalized Hooke's Law and is expressed as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad (i, j, k, l = 1,2,3) \tag{1}$$

And

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} \quad (i, j, k, l = 1,2,3) \tag{2}$$

where σ_{ij} and ε_{ij} are stress and strain tensors respectively, C_{ijkl} is the stiffness matrix and S_{ijkl} is the compliance matrix.

Generalized Hooke's Law can also be expressed in terms of contracted notation as

$$\sigma_i = C_{ij}\varepsilon_j \quad (i, j = 1,2,...,6) \tag{3}$$

and

$$\varepsilon_i = S_{ij}\sigma_j \quad (i, j = 1,2,...,6) \tag{4}$$

This notation is obtained by letting

$$\begin{array}{ll} \sigma_{11} = \sigma_1 & \varepsilon_{11} = \varepsilon_1 \\ \sigma_{22} = \sigma_2 & \varepsilon_{22} = \varepsilon_2 \\ \sigma_{33} = \sigma_3 & \varepsilon_{33} = \varepsilon_3 \\ \sigma_{23} = \sigma_4 = \tau_4 = \tau_{23} & 2\varepsilon_{23} = \gamma_{23} = \varepsilon_4 \\ \sigma_{31} = \sigma_5 = \tau_5 = \tau_{31} & 2\varepsilon_{31} = \gamma_{31} = \varepsilon_5 \\ \sigma_{12} = \sigma_6 = \tau_6 = \tau_{12} & 2\varepsilon_{12} = \gamma_{12} = \varepsilon_6 \end{array} \tag{5}$$

and

$$C_{1111} = C_{11},\ C_{1122} = C_{12};\ C_{1133} = C_{13};\ C_{1123} = 2C_{14};\ C_{1131} = 2C_{15};\ C_{1112} = 2C_{16};\ \text{etc.}$$

Both stiffness and compliance matrices are symmetric and the compliance matrix is the inverse of the stiffness matrix. These matrices contain 81 elastic constants which reduce to 36 due to the symmetry of stress and strain tensors and further reduce to 21 due to the symmetry of stiffness and compliance matrices. Hence for a general anisotropic material a total of 21 elastic constants are needed to fully characterize the material. Materials may exhibit planes of material symmetry which in turn reduces the number of elastic constants needed to characterize the material. The direction perpendicular to the plane of material symmetry is known as the principal material axes. Based on the number of planes of material symmetry that a material possesses, the materials are classified as follows:

Monoclinic materials: These materials contain one plane of material symmetry and the number of elastic constants needed is 13.

Orthotropic materials: Two mutually orthogonal planes of material symmetry exist and the number of elastic constants needed to characterize the material reduces to nine. Note that if there are two planes of material symmetry, then a third plane of material symmetry exists which is mutually orthogonal to the other two.

Transversely isotropic materials: These materials have one plane of symmetry and the number of elastic constants reduces to five.

Isotropic materials: Every plane is a plane of material symmetry and only two elastic constants are needed to characterize the material.

The fiber-reinforced polymers (FRP) are in general considered as orthotropic or transversely isotropic. The higher stiffness composites manufactured from unidirectional lamina are orthotropic and the short fiber composites are in general transversely isotropic. In this chapter the emphasis will be on the experimental characterization of the unidirectional composites. The methods that will be presented are based on the testing standards specified by American Society for Testing and Materials (ASTM).

For an orthotropic material the stress-strain relationship with respect to its principal material directions is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} \tag{6}$$

or

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} \tag{7}$$

The compliance terms in terms of the engineering constants are expressed as

$$\begin{aligned} & S_{11} = \frac{1}{E_1} \quad S_{12} = S_{21} = -\frac{\nu_{21}}{E_2} = -\frac{\nu_{12}}{E_1} \\ & S_{22} = \frac{1}{E_2} \quad S_{13} = S_{31} = -\frac{\nu_{31}}{E_3} = -\frac{\nu_{13}}{E_1} \\ & S_{33} = \frac{1}{E_3} \quad S_{23} = S_{32} = -\frac{\nu_{32}}{E_3} = -\frac{\nu_{23}}{E_2} \\ & S_{44} = \frac{1}{G_{23}} \; S_{55} = \frac{1}{G_{13}} \; S_{66} = \frac{1}{G_{12}} \end{aligned} \tag{8}$$

where E_i $(i = 1,2,3)$ are the moduli of elasticity associated with the principal material directions, ν_{ij} $(i, j = 1,2,3)$ are the Poisson's ratio (e.g. $\nu_{12} = -\varepsilon_2 / \varepsilon_1$ is obtained from a uniaxial test when the applied stress is in the 1-direction), and $G_{i,j}$ $(i, j = 1,2,3)$ are the shear moduli associated with the plane ij .

Hygrothermal strains due to change in moisture and temperature can be evaluated as

$$\begin{bmatrix} \varepsilon_1^{MT} \\ \varepsilon_2^{MT} \\ \varepsilon_3^{MT} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \Delta T + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \Delta c \tag{9}$$

where ΔT is the change in temperature, Δc is the change in moisture content, α_i $(i = 1,2,3)$ are the thermal expansion coefficients and β_i $(i = 1,2,3)$ are the moisture expansion coefficients. Note that the shear strains are zero due to the hygrothermal effects.

2.1.1. Constitutive Relationships for a Unidirectional Lamina

The building block in a typical laminated composite is the unidirectional lamina. Once a unidirectional lamina is characterized, then the properties of laminates with different architecture can be deduced relatively easily.

The thickness of a unidirectional lamina is very small compared to its in-plane dimensions, hence for all practical purposes a state of plane stress can be assumed. If $\sigma_3 = 0$, $\tau_{23} = \tau_4 = 0$ and $\tau_{13} = \tau_5 = 0$ then the stress strain relations reduce to

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} \tag{11}$$

where Q_{ij} are the reduced stiffness coefficients and they are given by

$$\begin{aligned} Q_{11} &= C_{11} - \frac{C_{13}C_{13}}{C_{33}} = \frac{E_1}{1-\nu_{12}\nu_{21}} \\ Q_{12} &= C_{12} - \frac{C_{13}C_{23}}{C_{33}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \\ Q_{22} &= C_{22} - \frac{C_{23}C_{23}}{C_{33}} = \frac{E_2}{1-\nu_{12}\nu_{21}} \\ Q_{66} &= C_{66} = G_{12} \end{aligned} \tag{12}$$

The compliance terms S_{ij} are as before given by Eqs. (8).

2.1.1.1 Transformation of Stress-Strain Relationships for Unidirectional Lamina

If the lamina is loaded in a direction other than the principal material directions, it is more advantages to express the stress-strain relationship with respect to a coordinate system more suitable to handle the problem in hand.

Consider that the x-y coordinate system is oriented with respect to the principal axes 1-2 as shown in Figure 1. Then the stresses in the 1-2 and x-y coordinate systems are related to each other through

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_6\end{bmatrix} = \mathbf{T}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} \text{ with } \mathbf{T} = \begin{bmatrix} m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2-n^2\end{bmatrix} \tag{13}$$

$$\text{and} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \mathbf{T}^{-1}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_6\end{bmatrix} \text{ where } \mathbf{T}^{-1} = \begin{bmatrix} m^2 & n^2 & -2mn\\ n^2 & m^2 & 2mn\\ mn & -mn & m^2-n^2\end{bmatrix} \tag{14}$$

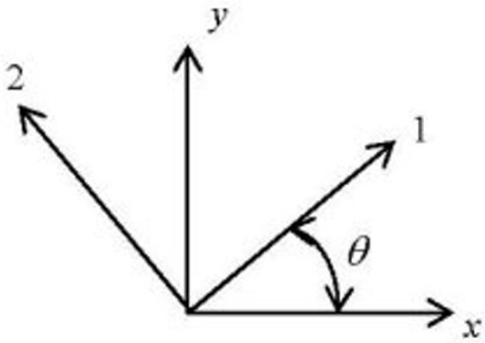

Figure 1. Coordinate system for transformation between the principal material axes and the x-y coordinate system

Note that $\mathbf{T}^{-1}$ is the inverse of the transformation matrix $\mathbf{T}$ and, $m = \cos(\theta)$ and $n = \sin(\theta)$.

The strain transformations can be achieved through the use of the transformation matrix and/or its inverse, e.g.

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_6\end{bmatrix} = \mathbf{T}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \frac{1}{2}\gamma_{xy}\end{bmatrix} \text{ and } \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \frac{1}{2}\gamma_{xy}\end{bmatrix} = \mathbf{T}^{-1}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_6\end{bmatrix} \tag{15}$$

Using the transformation relationships Eqs. (12)-(14) together with Eq. (10) we can write

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \mathbf{T} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} \tag{16}$$

which is reduced to

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \tag{17}$$

where

$$\begin{aligned}
\overline{Q}_{11} &= m^4 Q_{11} + n^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\
\overline{Q}_{22} &= n^4 Q_{11} + m^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\
\overline{Q}_{12} &= m^2 n^2 Q_{11} + m^2 n^2 Q_{22} + (m^4 + n^4) Q_{12} - 4m^2 n^2 Q_{66} \\
\overline{Q}_{16} &= m^3 n Q_{11} - mn^3 Q_{22} + (mn^3 - m^3 n) Q_{12} + 2(mn^3 - m^3 n) Q_{66} \\
\overline{Q}_{26} &= mn^3 Q_{11} - m^3 n Q_{22} + (m^3 n - mn^3) Q_{12} + 2(m^3 n - mn^3) Q_{66} \\
\overline{Q}_{66} &= m^2 n^2 Q_{11} + m^2 n^2 Q_{22} - 2m^2 n^2 Q_{12} + (m^2 - n^2)^2 Q_{66}
\end{aligned} \tag{18}$$

Note that in the *x*-*y* coordinate system, although the material is orthotropic, this is not evident from Eq. (17) since the transformed stiffness matrix $\overline{Q}_{ij}$ has all non-zero coefficients, the same as an anisotropic material. The coefficients given in Eq. (18) are not independent; they are all functions of the four lamina stiffnesses with respect to the principal material axes.

Alternatively the strains are expressed in the *x*-*y* coordinate system as

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} & \overline{S}_{16} \\ \overline{S}_{21} & \overline{S}_{22} & \overline{S}_{26} \\ \overline{S}_{61} & \overline{S}_{61} & \overline{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \tag{19}$$

where the transformed lamina compliances $\overline{S}_{ij}$ are

$$\begin{aligned}
\overline{S}_{11} &= m^4 S_{11} + n^4 S_{22} + 2m^2 n^2 S_{12} + m^2 n^2 S_{66} \\
\overline{S}_{22} &= n^4 S_{11} + m^4 S_{22} + 2m^2 n^2 S_{12} + m^2 n^2 S_{66} \\
\overline{S}_{12} &= m^2 n^2 S_{11} + m^2 n^2 S_{22} + (m^4 + n^4) S_{12} - m^2 n^2 S_{66} \\
\overline{S}_{16} &= m^3 n S_{11} - 2mn^3 S_{22} + 2(mn^3 - m^3 n) S_{12} + (mn^3 - m^3 n) S_{66} \\
\overline{S}_{26} &= mn^3 S_{11} - 2m^3 n S_{22} + 2(m^3 n - mn^3) S_{12} + (m^3 n - mn^3) S_{66} \\
\overline{S}_{66} &= 4m^2 n^2 S_{11} + 4m^2 n^2 S_{22} - 8m^2 n^2 S_{12} + (m^2 - n^2)^2 S_{66}
\end{aligned} \tag{20}$$

In terms of engineering constants, the stress strain relationship becomes

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1/E_x & -\nu_{yx}/E_y & \eta_{xy,x}/G_{xy} \\ -\nu_{xy}/E_x & 1/E_y & \eta_{xy,y}/G_{xy} \\ \eta_{x,xy}/E_x & \eta_{y,xy}/E_y & 1/G_{xy} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \tag{21}$$

From the symmetry of the compliance matrix we get

$$\begin{aligned} \frac{\nu_{xy}}{E_x} &= \frac{\nu_{yx}}{E_y} \\ \frac{\eta_{xy,x}}{G_{xy}} &= \frac{\eta_{x,xy}}{E_x} \\ \frac{\eta_{xy,y}}{G_{xy}} &= \frac{\eta_{y,xy}}{E_y} \end{aligned} \tag{22}$$

The engineering constants are determined from simple uniaxial tests. For example if $\sigma_{xx} \neq 0$ and all the other stress components are zero, then $E_x = \sigma_{xx}/\varepsilon_{xx}$ is the modulus of elasticity associated with the *x*-direction, $\nu_{xy} = -\varepsilon_{yy}/\varepsilon_{xx}$ is the Poisson's ratio, and $\eta_{x,xy} = \gamma_{xy}/\varepsilon_{xx}$ is the shear coupling coefficient. Likewise if $\sigma_{yy} \neq 0$ and all the other stress components are zero, then $E_y = \sigma_{yy}/\varepsilon_{yy}$ is the modulus of elasticity associated with the *y*-direction, $\nu_{yx} = -\varepsilon_{xx}/\varepsilon_{yy}$, and $\eta_{y,xy} = \gamma_{xy}/\varepsilon_{yy}$, and if $\tau_{xy} \neq 0$ and all the other stress components are zero, then $G_{xy} = \tau_{xy}/\gamma_{xy}$ is the shear modulus associated with the *x*-*y* directions, $\eta_{xy,x} = \varepsilon_{xx}/\gamma_{xy}$ and, $\eta_{xy,y} = \varepsilon_{yy}/\gamma_{xy}$.

2.1.2. Constitutive Relationships for a Laminate

Composite materials are used by forming a laminate from individual lamina. Each lamina is thin and may have different fiber orientation and in some cases may have different materials (see Figure 2). The response of the laminates to loads depends on the properties, fiber orientation and stacking sequence of its layers. Two laminates composed of identical lamina, with the same thickness will respond to loads differently if laminae are arranged differently. Analysis of laminates is mostly restricted to flat panels with limited discussion on curved laminates which can be found in textbooks on mechanics of composite materials.

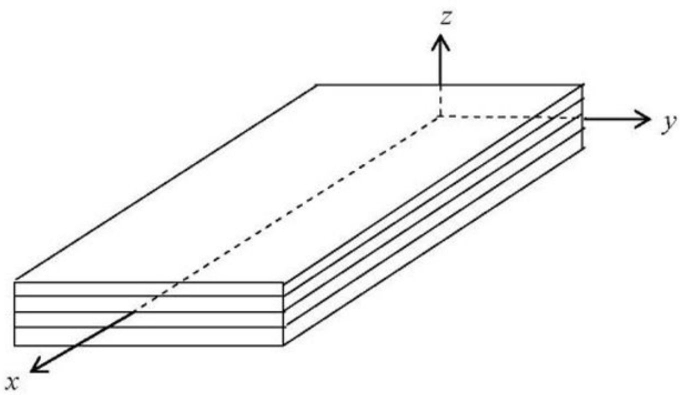

Figure 2. Laminated structure

Analysis of flat laminates under in-plane and bending loads are based on the Kirchoff-Love hypothesis. Assuming that all layers are perfectly bonded (no slippage between the layers), Kirchoff hypothesis state that a line perpendicular to geometric midsurface (reference plane) remains straight and does not change its length while the laminate is deforming under the applied loading. This hypothesis results in $\varepsilon_{zz} = 0$, $\gamma_{xz} = \gamma_{yz} = 0$. Then the strain distribution in the laminate is expressed by

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{23}$$

where *z* is measured from the geometric midsurface as shown in Figure 3. The strain components $\varepsilon_{xx}^0, \varepsilon_{yy}^0, \gamma_{xy}^0$ are the strains of the geometric midsurface (reference plane) and $\kappa_x, \kappa_y, \kappa_{xy}$ are the bending curvatures of the reference plane in the *xz*, *yz* and *xy* planes respectively.

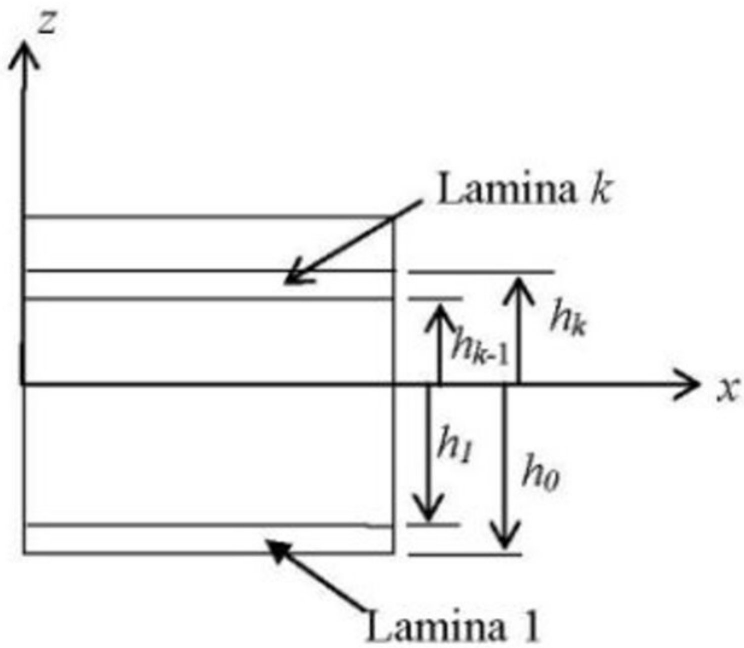

Figure 3. Laminate with coordinate notation for individual lamina

Determination of the force, N_x, N_y, N_{xy} and moment, M_x, M_y, M_{xy} resultants yields the following constitutive relationships

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{25}$$

where

$$A_{ij} = \sum_{k=1}^{n} \overline{Q}_{ij} (h_k - h_{k-1}) \qquad i, j = 1,2,6 \quad \text{(Extensional Stiffness)}$$

$$B_{ij} = \frac{1}{2} \sum \overline{Q}_{ij} (h_k^2 - h_{k-1}^2) \qquad i, j = 1,2,6 \quad \text{(Coupling Stiffness)}$$

$$D_{ij} = \frac{1}{3} \sum \overline{Q}_{ij} (h_k^3 - h_{k-1}^3) \qquad i, j = 1,2,6 \quad \text{(Bending Stiffness)}$$

If temperature and moisture change is considered, then the force and moment results are

$$\mathbf{N}_{xy}^{\mathrm{MT}} = \sum_{k=1}^{n} \bar{\mathbf{Q}}_{ij}^{k} \boldsymbol{\alpha}_{xy}^{k} (h_k - h_{k-1}) \Delta T + \sum_{k=1}^{n} \bar{\mathbf{Q}}_{ij}^{k} \boldsymbol{\beta}_{xy}^{k} (h_k - h_{k-1}) \Delta c \tag{26}$$

$$\mathbf{M}_{xy}^{\mathrm{MT}} = \sum_{k=1}^{n} \frac{1}{2} \bar{\mathbf{Q}}_{ij}^{k} \boldsymbol{\alpha}_{xy}^{k} (h_k^2 - h_{k-1}^2) \Delta T + \sum_{k=1}^{n} \frac{1}{2} \bar{\mathbf{Q}}_{ij}^{k} \boldsymbol{\beta}_{xy}^{k} (h_k^2 - h_{k-1}^2) \Delta c \tag{27}$$

and these can be superimposed to the force and moment resultants given in Eqs. (24) and (25).

Load-deformation relationships can be inverted to express strains and curvatures in terms of loads and moments, i.e.,

$$\begin{bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{bmatrix} = \mathbf{A}' \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \mathbf{B}' \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} \tag{28}$$

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \mathbf{C}' \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \mathbf{D}' \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} \tag{29}$$

where the matrices $\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}'$ and $\mathbf{D}'$ are computed as

$\mathbf{A}' = \mathbf{A}^{-1} - \mathbf{B}^{*}\mathbf{D}^{*-1}\mathbf{C}^{*}$

$\mathbf{B}' = \mathbf{B}^{*}\mathbf{D}^{*-1}$

$\mathbf{C}' = -\mathbf{D}^{*-1}\mathbf{C}^{*}$

$\mathbf{D}' = \mathbf{D}^{*-1}$

with

$\mathbf{B}^{*} = -\mathbf{A}^{-1}\mathbf{B}$

$\mathbf{C}^{*} = \mathbf{B}\mathbf{A}^{-1}$

$\mathbf{D}^{*} = \mathbf{D} - \mathbf{B}\mathbf{A}^{-1}\mathbf{B}$

it should be noted that $\mathbf{B}' \neq \mathbf{C}'$

What are given above are the main constitutive equations for a single lamina and laminates. More detailed analysis of lamina and laminates can be found in the textbooks on mechanics of composite materials.

3. Characterization of Composite Material Properties

A lamina is the main building block of a laminate, and the properties of a lamina along with the stacking sequence of the lamina are used to predict the response of laminates to loading. These predictions need the basic properties of the constituents of the lamina and the lamina itself. Constituent properties are needed to "design" the lamina through micromechanical models that relate the constituent properties to the property of a lamina. Of course experimental measurement of properties is needed at both the constituent and lamina levels.

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Biographical Sketch

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